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**Georgia Performance Standards Framework for MATHEMATICS – GRADE 8**

**Unit Five Organizer: “SLIPPERY SLOPE”  
(5 weeks)**

**OVERVIEW:**

In this unit students use symbolic algebra to represent situations and solve problems, especially those that involve linear relationships. They will also use equations, tables and graphs to analyze and interpret linear functions, and make inferences from statistical data, particularly data that can be modeled by linear functions. The material in this unit should extend those algebraic concepts begun in sixth grade and prepare students for the more specific and in-depth treatment of systems of linear equations and inequalities presented in Unit 7.

In this unit students will:

- collect data that occurs as a result of relationships between varying quantities;
- organize collected data into tables;
- graph data that occurs as a result of relationships between varying quantities in the coordinate plane;
- analyze graphs, tables and equations to determine the relationship between varying quantities;
- interpret slope as how the rate of change in one variable affects the other;
- determine the meaning of slope and  $y$ -intercept in a given situation;
- graph equations in both slope-intercept and standard form;
- identify functions as linear or nonlinear;
- graph the open or closed half-plane solution set of a linear inequality;
- solve problems involving linear relationships by gathering data, graphing the data as a scatter plot, determining the line of best fit, writing its equation, and interpreting the solution of the equation in the context of the original problem.

To assure that this unit is taught with the appropriate emphasis, depth and rigor, it is important that the tasks listed under “Evidence of Learning” be reviewed early in the planning process. A variety of resources should be utilized to supplement, but not completely replace, the textbook. Textbooks not only provide much needed content information, but excellent learning activities as well. The tasks in this unit illustrate the type of learning activities that should be utilized from a variety of sources.

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**ENDURING UNDERSTANDINGS:**

- Collecting and examining data can sometimes help one discover patterns in the way in which two quantities vary.
- Changes in varying quantities are often related by patterns which, once discovered, can be used to predict outcomes and solve problems.
- Written descriptions, tables, graphs and equations are useful in representing and investigating relationships between varying quantities.
- Different representations (written descriptions, tables, graphs and equations) of the relationships between varying quantities may have different strengths and weaknesses.
- Linear functions may be used to represent and generalize real situations.
- Slope and  $y$ -intercept are keys to solving real problems involving linear relationships.

**ESSENTIAL QUESTIONS:**

- What does the data tell me?
- How does a change in one variable affect the other variable in a given situation?
- Which tells me more about the relationship I am investigating – a table, a graph or an equation? Why?
- What strategies can I use to help me understand and represent real situations involving linear relationships?
- How can the properties of lines help me to understand graphing linear functions?
- How is a linear inequality like a linear equation? How are they different?

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**STANDARDS ADDRESSED IN THIS UNIT**

**Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.**

**KEY STANDARDS:**

**Students will use linear algebra to represent, analyze and solve problems. They will use equations, tables, and graphs to investigate linear relations and functions, paying particular attention to slope as a rate of change. They will make inferences from data that can be modeled by linear functions.**

**M8A3. Students will understand relations and linear functions.**

- d. Recognize functions in a variety of representations and a variety of contexts.
- h. Identify relations and functions as linear or nonlinear.
- i. Translate among verbal, tabular, graphic, and algebraic representations of functions.

**M8A4. Students will graph and analyze graphs of linear equations and inequalities.**

- a. Interpret slope as a rate of change.
- b. Determine the meaning of the slope and y-intercept in a given situation.
- c. Graph equations of the form  $y = mx + b$ .
- d. Graph equations of the form  $ax + by = c$ .
- e. Graph the solution set of a linear inequality, identifying whether the solution set is an open or a closed half-plane.
- f. Determine the equation of a line given a graph, numerical information that defines the line or a context involving a linear relationship.
- g. Solve problems involving linear relationships.

**M8D4. Students will organize, interpret, and make inferences from statistical data.**

- a. Gather data that can be modeled with a linear function.
- b. Estimate and determine a line of best fit from a scatter plot.

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**RELATED STANDARDS:**

**M8P1. Students will solve problems (using appropriate technology).**

- a. Build new mathematical knowledge through problem solving.
- b. Solve problems that arise in mathematics and in other contexts.
- c. Apply and adapt a variety of appropriate strategies to solving problems.
- d. Monitor and reflect on the process of mathematical problem solving.

**M8P2. Students will reason and evaluate mathematical arguments.**

- a. Recognize reasoning and proof as fundamental aspects of mathematics.
- b. Make and investigate mathematical conjectures.
- c. Develop and evaluate mathematical arguments and proofs.
- d. Select and use various types of reasoning and methods of proofs.

**M8P3. Students will communicate mathematically.**

- a. Organize and consolidate their mathematical thinking through communication.
- b. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- c. Analyze and evaluate the mathematical thinking and strategies of others.
- d. Use the language of mathematics to express mathematical ideas precisely.

**M8P4. Students will make connections among mathematical ideas and to other disciplines.**

- a. Recognize and use connections among mathematical ideas.
- b. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- c. Recognize and apply mathematics in contexts outside of mathematics.

**M8P5. Students will represent mathematics in multiple ways.**

- a. Create and use representations to organize, record, and communicate mathematical ideas.
- b. Select, apply, and translate among mathematical representations to solve problems.
- c. Use representations to model and interpret physical, social, and mathematical phenomena.

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**CONCEPTS/SKILLS TO MAINTAIN:**

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Simplifying and evaluating algebraic expressions
- Translating verbal phrases to algebraic expressions
- Solve one- and two-step equations
- Plot points on a coordinate plane
- Add, subtract, multiply, and divide rational numbers

**SELECTED TERMS AND SYMBOLS:**

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

**Arithmetic sequence:** A sequence of numbers in which the difference between any two consecutive terms is the same

**Constant function:** A function that is written  $y = k$ , where  $k$  is a real number. The  $y$  value is constant for all values of  $x$ . The graph of a constant function is a horizontal line.

**Function:** A relation (set of ordered pairs) such that each  $x$  value is associated with only one  $y$  value.

**Graph of a linear inequality:** The solutions of a linear inequality, forming a half-plane on one side of a line and may or may not also form the line itself.

**Half-plane:** The portion of a plane on one side of a line.

**Line of best fit:** The line that best represents the trend established by the points in a particular scatter plot.

**Point-slope form:** Derived from the fact that if one point on a line and the slope of that same line are known, the line may be determined or drawn,  $y - y_1 = m(x - x_1)$

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**Scatter plot:** The graph of a collection of ordered pairs.

**Slope:** The steepness of a line, which may be calculated by finding the ratio of the difference between the  $y$  values of two points on the line to the difference between the corresponding  $x$  values of those two points on the line.

**Slope-intercept form:** One way to write an equation of a line; uses the form  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept.

**Standard form:** Also known as ‘General form’ for a linear equation in two variables,  $x$  and  $y$ .

It is usually given as  $Ax + By = C$  where, if at all possible,  $A$ ,  $B$ , and  $C$  are integers, and  $A$  is non-negative, and,  $A$ ,  $B$ , and  $C$  have no common factors other than 1.

You may visit <http://intermath.coe.uga.edu> and click on dictionary to see definitions and specific examples of terms and symbols used in the eighth grade GPS.

**EVIDENCE OF LEARNING:**

**By the conclusion of this unit, students should be able to demonstrate the following competencies:**

- use tables of values to describe sequences with a formula in closed form;
- find the  $n^{\text{th}}$  term of an arithmetic sequence;
- determine whether a function is linear or nonlinear;
- translate among different representations of functions (verbal descriptions, tables, graphs, and equations);
- interpret slope as a rate of change;
- explain the meaning of the slope and  $y$ -intercept in a problem situation;
- graph equations in the form  $y = mx + b$  and  $ax + by = c$ ;
- find an equation of a line given the graph, numerical information (such as two points on the line or the slope and one point on the line), or a problem context; and
- find the line of best fit for a set of data and use the line of best fit to make predictions.

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The following tasks represent the level of depth, rigor, and complexity expected of all 8<sup>th</sup> grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning.

- Heartbeats
- Drain the Pool Learning Task
- Faster Than a Graphing Calculator
- Forget the Formula
- Walk the Graph
- Mineral Samples
- What is My Ring Size?
- Right to the Point Learning Task
- Staircases
- Crossing the River
- What Would That Graph Look Like?
- Cholesterol – Good or Bad?
- Them Bones
- Bungee Jump
- Is the Data Linear?

**Culminating Activity:** “Is the Data Linear?”

In this task students will gather data for several experiments and determine whether the data relationships are linear. For experiments which represent linear relationships, students will find the lines of best fit and interpret the meaning of the slope and  $y$ -intercept in relation to the manipulated and responding variables.

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**STRATEGIES FOR TEACHING AND LEARNING:**

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols and words.
- Appropriate manipulatives and technology should be used to enhance student learning.
- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.

**TASKS:**

The collection of the following tasks represents the level of depth, rigor and complexity expected of all eighth-grade students to demonstrate evidence of learning.

- **Heartbeats**



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## Heartbeats

In the sixth and seventh grade, you studied about proportional relationships. Do you think that the number of heartbeats you can count is proportional to the number of seconds that you check your pulse? Explain why or why not.

Work with your partner to take measurements and test your conjecture. One of you will be the timer and the other will count their own number of heart beats per period of time. First, count and record the number of beats in 10 seconds, and repeat the experiment counting the number of beats in 20 seconds. Repeat again for 40 seconds.

After gathering this data, change jobs. The person who kept time now checks his/her pulse rate for 10 seconds, 20 seconds, and 40 seconds.

Predict how many times your heart would beat in 25 seconds, in 60 seconds, and in 120 seconds. Explain how you made your predictions.

Develop an equation to represent your pulse rate. How does your equation compare with your partner's equation? Explain to your partner why your equation is valid.

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### ***Discussion, Suggestions, Possible Solutions***

*In “Heartbeats” students should solidify the connection between direct proportions and linear equations. Their initial explanations of the number of heartbeats being directly proportional to the number of seconds could center on arguments such as, “If I checked my pulse three times as long, then the number of beats should be three times as many.”*

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*Most students will probably want to start by organizing their data in a table similar to the one below.*

Number of seconds	Number of Heartbeats
10	
20	
25	
40	
60	
120	

*Students will use a variety of different methods to determine the number of heartbeats for 25, 60, and 120 seconds. Some may find the number of heartbeats for 25 seconds by adding half of the number of heartbeats collected for 10 seconds to the number of heartbeats collected for 20 seconds. Others may add the average of the number of heartbeats collected for 10 and 20 seconds to the number of heartbeats for 10 seconds. To find the number of heartbeats for 60 seconds, some students may triple the number of heartbeats for 20 seconds while others may add the number of heartbeats for 20 and 40 seconds. There will also be students that graph the data and use the graph to make predictions. As long as the students can estimate the number of heartbeats using correct mathematical thinking, encourage them to be creative.*

*In developing an equation using the collected data, some may experience the fact that not all of their data will be linear. This could take the class into a discussion of irregular heartbeats and using the data collected to determine if their heartbeats were regular. Later in the unit, the teacher may elect to return to this task and ask students to find a line of best fit for any non-linear data. Students could come up with possible explanations for variations in heart-rate, such as the effect of exercise, health conditions (thyroid problems, e.g.), etc.*

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*Direct proportions are one class of linear equations. In a direct proportion, the y-intercept is 0. Thus, direct proportions are of the form  $y = kx$  as studied in the sixth and seventh grades.*

*To extend the heartbeat activity, pose the question of how to interpret  $y = 72x$  if it represents an equation describing the kind of data involved in this problem situation. (If  $x$  is the number of minutes and the heart beats 72 times per minute, then the total number of beats  $y$  is  $72x$ .) Emphasize the meaning of the slope as the rate of change: for each additional minute the number of heartbeats increases by 72. Pose the hypothetical situation that another person has  $y = 120x$  as the equation for her number of heartbeats. Ask the students to interpret what this means. The discussion would certainly include the fact that this woman's heart rate is 120 beats per minute, which is much greater than the other rate of 72 beats per minute.*

*Teachers should support good student dialogue and take advantage of comments and questions to help guide them into correct mathematical thinking.*

- **Drain the Pool Learning Task**

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### Drain the Pool Learning Task

Used with permission from "Using Student Work to Develop Teachers' Knowledge of Algebra" by Beth A. Herbel-Eisenmann and Elizabeth Difanis Phillips, **Mathematics Teaching in the Middle School**, September 2005, page 63.  
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Suppose you turn on a pump and let it run to empty the water out of a pool. The amount of water in the pool ( $W$ , measured in gallons) at any time ( $T$ , measured in hours) is given by the following equation:  
 $W = -350(T - 4)$ .

Answer each of the questions below and explain how you used the equation to do so.

- How many gallons of water are being pumped out each hour?
- How much water was in the pool when the pumping started?
- How long will it take for the pump to empty the pool completely?
- Write an equation that is equivalent to  $W = -350(T - 4)$ . What does this second equation tell you about the situation?
- Describe what the graph of the relationship between  $W$  and  $T$  looks like.

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### Discussion, Suggestions, Possible Solutions

*In "Drain the Pool Learning Task" the students are given an equation and the context for its meaning. From that context they are to interpret the meaning of the constants in the equation. The rate of change is -350, meaning that for each hour that passes, the volume is going down by 350 gallons. In order for the amount of water to be zero, which means the pool is drained completely,  $-350(T - 4) = 0$ . Solving this equation yields  $T = 4$ . Thus, it takes 4 hours for the pool to drain completely. Originally ( $T = 0$ ), the pool held  $-350(-4) = 1400$  gallons of water. The equivalent form of the given equation which is obtained by using the distributive property is  $W = -350T + 1400$ . In this form it is apparent that the original volume was 1400 gallons and that the rate of change is a loss of 350 gallons per minute. The graph has a y-intercept of 1400 and a slope of -350.*

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More depth:

*Once students have a good understanding of how to develop the meaning of an equation in context, the teacher may increase student understanding by giving a learning task that reverses their thinking. Below is an example of a learning task that requires students to develop an equation based on the context of a situation.*

*The Student Council is going to buy equipment to make thingamajigs to sell to raise money for a school project. The equipment will cost \$400, regardless of how many thingamajigs are produced. In addition to the initial equipment purchase, the Student Council will have a cost of \$7 for each thingamajig. Write an equation for the total cost for producing  $x$  thingamajigs. How much will it cost to produce 80 thingamajigs?*

*In “Drain the Pool Learning Task” the students were given the equation and were asked to interpret the meaning of the equation in light of the problem context. In this situation, the students use their conceptual understanding of the coefficients of a linear equation to write the appropriate equation. Since the cost of the equipment is \$400, even if zero thingamajigs are sold, the y-intercept is 400. The total cost rises \$7 for each additional thingamajig, so the rate of change (slope) is \$7.*

*Thus, the equation for the total cost is  $C = 7x + 400$ . The cost of 80 thingamajigs would be  $7(80) + 400 = \$960$ .*

*To assess understanding of proportion, the teacher might ask whether the cost of 160 thingamajigs would be twice as much as the cost of 80. This is not true because of the role of the initial fixed cost of \$400.*

More depth and rigor:

*In developing more depth and rigor of understanding, students should be able to write equations for situations that do not involve any actual numbers such as the following learning task.*

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*A company that produces pens has  $n$  pens in stock at the beginning of a certain day. It produces these pens at a constant rate,  $r$ , for the entire day. Write an equation to illustrate this situation. If a new machine has been purchased and pens are now being produced at a greater constant rate, write a new equation that can be used to determine the number of pens produced during a day. Discuss the characteristics of this new situation comparing and contrasting the two different scenarios.*

*In this context, students should be able to discern that this information is linear and the rate,  $r$ , for both days would represent the slope of a line while the number of pens in stock at the beginning of a that day,  $n$ , would represent the y-intercept. They should be able to describe what would happen to the slope of the line in the new situation relative to the slope of the line in the original situation.*

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*A direct link to this article is [http://my.nctm.org/eresources/view\\_media.asp?article\\_id=7157](http://my.nctm.org/eresources/view_media.asp?article_id=7157).*

- **Faster than a Graphing Calculator**

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## Faster than a Graphing Calculator

### Part I:

Matt and Tyler have been studying graphing linear equations. Matt challenged Tyler to a race to see who could graph  $y = 3x + 5$  in the least amount of time. Matt is going to graph the equation by hand, and Tyler is going to use the graphing calculator. Matt says he can graph the equation in less than ten seconds, in less time than Tyler can enter the equation in the calculator and press the “Graph” key. Explain how you think Matt intends to graph the equation. Illustrate his method on two more linear equations that you make up to give to Matt and Tyler to use for their race.

### Part II:

Who do you think will win the race if Matt and Tyler are given the equation  $3x + 4y = 12$  to graph? Why? How do you predict Matt will graph the equation?

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### *Discussion, Suggestions, Possible Solutions*

#### Part I:

*Regardless of whether students have used graphing calculators to graph linear equations, they should be able to explain how an equation in slope-intercept form may be graphed quickly by first locating the y-intercept and then using the rise and run to locate another point on the line. In the example given, the student should explain that Matt will locate the y-intercept (0, 5), and then go over one unit and up three to locate another point on the line. These two points are then used to sketch the line.*

*In posing other examples for Matt and Tyler, elicit different variations, such as negative slopes, negative y-intercepts, constants which lend themselves to using intervals of ten between tic marks, etc. (Ex.  $y = -30x - 20$ )*

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*Students might also respond that Matt could quickly get a table of values. Using  $x = 0$ , Matt has the y-intercept of 5. Using  $x = 1$ , Matt obtains the point (1, 8).*

Part II:

*If Matt chooses to get the x- and y-intercepts to sketch the line, the arithmetic can be done mentally to obtain (0, 3) and (4, 0).*

*Students may decide to use the slope-intercept form because it is required to enter as 'y =' on the TI-83 calculator. While changing the equation to slope-intercept is certainly acceptable, students should be encouraged to think about which method is preferable in a particular situation, rather than blindly using the same method each time. This is particularly important when students are graphing systems of inequalities and later doing linear programming problems. In linear programming, for example, having the intercepts is helpful in locating the vertices of the feasible region. In order to teach for transfer, teachers must understand how the skills will be utilized in more complex problems.*

*When using the Standard Form of an equation, they may feel that A, B, and C do not stand for something obvious. However, should this form be transformed to the more popular slope-intercept form, students may see the relationship upon noticing  $y = \frac{-A}{B}x + \frac{C}{B}$ . Once students are familiar with this relationship, it will be much easier for them to transfer from one form to the other.*

*For future use of the Standard Form of an equation, teachers should understand this form fits well with a vector formulation of geometry. Using vectors, this may be written as a "dot product"  $(A,B) \cdot (x, y) = C$ . This form of the equation of a line also relates to "linear combination" in more advance mathematics.*



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- **Forget the Formula**

### **Forget the Formula**

Mrs. Howell, your science teacher, overheard two of her students talking about how to convert temperatures from Celsius to Fahrenheit and vice versa. The students said they knew there was a formula, but they didn't remember what it was. Mrs. Howell remarked to you that if they just knew about the freezing point and boiling point of water for each temperature scale, the formula could easily be "rediscovered." Mrs. Howell has asked you to write a written explanation for how to find the formula, showing all your calculations.

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#### ***Discussion, Suggestions, Possible Solutions***

*Students will have to remember or research to find that the freezing point of water is 0 degrees Celsius and 32 degrees Fahrenheit. The boiling point of water is 100 degrees Celsius and 212 degrees Fahrenheit.*

*One method of finding this formula is to use (0, 32) and (100, 212) as two points on a line. To find the equation of the line only the slope is needed, since the y-intercept (0, 32) is already given. Calculating the slope  $\frac{212-32}{100-0}$ , the students should simplify  $\frac{180}{100}$  to  $\frac{9}{5}$ . Substituting this for  $m$  in  $y = mx + b$ , the*

*equation becomes  $y = \left(\frac{9}{5}\right)x + 32$ . Because  $y$  is the Fahrenheit temperature and  $x$  is the Celsius*

*temperature, the formula would be more appropriately written  $F = \left(\frac{9}{5}\right)C + 32$ .*

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*Students could be encouraged to solve this equation for  $C$  to produce another form expressing the relationship:  $C = \frac{5}{9}(F - 32)$ .*

*They could also produce a graph of the corresponding temperatures.*

*The “Faster Than a Graphing Calculator” task deals with graphing a given linear equation and discerning which method to choose for graphing. “Forget the Formula” is representative of a variety of situations in which information is known about the graph and the students have to be able to find the equation from that information. In “Forget the Formula” the information used is two points on the given line; one of these happens to be the  $y$ -intercept if the Fahrenheit temperature is used as the  $y$  value. Be sure to include other variations on finding the equation of a line, such as given the slope and a point on the line which is not the  $y$ -intercept or two points on a line when one is not the  $y$ -intercept.*

*Should neither of the two points cross the  $y$ -axis, students would most likely use the point-slope form of an equation. This form is derived from finding the slope using two points on the line  $m = \frac{(y - y_1)}{(x - x_1)}$ . From this,*

*it is seen that  $y - y_1 = m(x - x_1)$  and  $y = m(x - x_1) + y_1$ . It is important for students to understand the connections between the various forms of linear equations.*

*An interactive website may be used to help students understand the relationships between the various forms of linear equations: <http://id.mind.net/~zona/mmts/functionInstitute/linearFunctions/linearFunctions.html>*

- **Walk the Graph**

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## Walk the Graph

Eddie's teacher used a motion detector hooked to an overhead graphing calculator to show graphs of how far Eddie was from the motion detector for a few seconds. Eddie was asked to walk in such a way as to produce graphs with certain characteristics. Explain how Eddie needs to walk to produce a graph which is:

- (1) A line with a negative slope.
- (2) A line with a positive slope.
- (3) A line with a steeper slope than the one in part (2).
- (4) A horizontal line.
- (5) Not a line.

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### *Discussion, Suggestions, Possible Solutions*

*Students will really enjoy the activity of actually walking the graphs after making their predictions if the technology is available. However, this task may still be used if the technology is not available.*

*When forming a graph, students should understand why time will be the independent variable and distance will be the dependent variable.*

*For graph (1), if the change in distance compared to the change in time is negative, then the walker must be getting closer to the motion detector as the time increases. Thus, the walker should stand as far away from the motion detector as possible but within range of the motion detector (staying in front of the motion detector) and walk at a steady pace towards the motion detector.*

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*For graph (2), Eddie should stand close to the motion detector when the timing begins and walk at a steady pace going away from the motion detector, yet staying in front of it.*

*For graph (3), Eddie repeats the process to produce graph (2), but walks at a faster pace. The distance away from the motion detector needs to be increasing more quickly than in graph (2).*

*For graph (4), Eddie may choose some distance away from the motion detector and remain in that position. This is an example of a constant function. His distance from the motion detector remains constant.*

*For graph (5), Eddie could vary the pace at which he walks. For example, he might start out as far away from the motion detector as possible, walk slowly at first, and then increase his speed until he gets very close to the motion detector. Then if he walks away from the detector, quickly at first, and then slowing down, he might even produce a graph that looks symmetrical. Teachers may have students predict what the graph will look like before they take this walk.*

- **Mineral Samples**

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### Mineral Samples

Last summer Ian went to the mountains and panned for gold. While he didn't find any gold, he did find some pyrite (fool's gold) and many other kinds of minerals. Ian's friend, who happens to be a geologist, took several of the samples and grouped them together. She told Ian that all of those minerals were the same. Ian had a hard time believing her, because they are many different colors. She suggested Ian analyze some data about the specimens. Ian carefully weighed each specimen in grams (g) and found the volume of each specimen in milliliters (ml).

Ian has asked you to be his science fair partner and help him analyze the data. Write your analysis of his data given below:

Specimen Number	Mass or weight (g)	Volume (ml)
1	17	7
2	10	4
3	13	5
4	16	6
5	7	3
6	24	10
7	5	2

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### *Discussion, Suggestions, Possible Solutions*

*Prior to asking students to participate in this task, teachers may want to revisit "Heartbeats" and make sure that students understand the purpose and procedures for establishing a line of best fit for a set of data.*

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*Students could graph the volume as the independent variable (x) and mass as the dependent variable (y). Their graphs could be produced either by hand on graph paper or as a statplot on a graphing calculator. Students could choose two points that are on their line of best fit that they determine by eye-balling the data, using a tool such as a raw spaghetti noodle, or they could use the linear regression feature of the calculator to obtain the regression equation  $y = 2.41x + .40$ .*

*Students might divide the mass by the volume by hand for each specimen and then find the average of this value. They could also have the calculator divide the values and find the average. The average value (2.49) is close to the coefficient of the x term in the regression equation. As students explore the meaning of this slope in this problem context, they should come to understand that it means every time the volume goes up by one ml the mass goes up by approximately 2.4 or 2.5 grams. This rate of mass in grams per milliliter of volume is the density of the mineral.*

*Research could be done to find lists of densities for particular minerals. While earth science references will list the density (also called “specific gravity”) of quartz as 2.6, the samples used for the data above were quartz. This could lead to a discussion of the precision and accuracy of measurements, as well as to a discussion of impurities and other factors that could influence the results.*

*The relationship between the mass and volume of the specimens might be described by some students as a proportional relationship. This could lead to the conclusion that a theoretical model for this relationship might be written as  $y = 2.6x$ . In other words, a hypothetical sample with a volume of zero would have a mass of zero, so the y-intercept should be zero.*

*As an extension, ask students where data points would be for samples of minerals that have a density greater than 2.6. These would be in the half-plane above the regression line for the quartz samples. This can provide a transition to graphing inequalities.*

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- **What is My Ring Size?**

### What is My Ring Size?

Courtney called Brittany and told her about finding a strip of paper in a catalogue to use for finding what ring size a person needs (See below.). Courtney wants to order Brittany a ring that is on sale, but Brittany doesn't have the paper to use to determine her ring size. Courtney describes the ring-sizing chart to Brittany: "A size 5 finger is 5.6 cm in circumference. For every additional 3 mm of circumference, the ring size increases by a whole size." Brittany says that sounds like a linear function to her. She tells Courtney good-bye and then sets out to find the equation for ring size. She then calls you and tells you to work it out and see if you come up with the same thing she does. When you call Brittany back, what will you tell her? Write the explanation that you will give to her about how you obtained your equation.



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### *Discussion, Suggestions, Possible Solutions*

*Students may use a variety of approaches to this task even beyond the ones mentioned below.*

One possible student approach:

*Let  $x$  be the number of millimeters above 5.6 for the circumference of the ring finger. Then  $\frac{x}{3}$  tells how many ring sizes above 5 the finger measures. Thus the ring size,  $y$ , will be  $5 + \frac{x}{3}$ . Using the commutative property, we have  $y = \frac{x}{3} + 5$ .*

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A second possible student approach:

*A student might also let  $x$  represent the circumference of the ring finger expressed in centimeters. In that case,  $y = \frac{x-5.6}{.3} + 5$ . This equation may be simplified:*

$$y = \frac{x}{.3} - \frac{5.6}{.3} + 5$$

$$y = \frac{10}{3}x - \frac{56}{3} + \frac{15}{3}$$

$$y = \frac{10}{3}x - \frac{41}{3}$$

A third possible student approach:

*Another approach would be to look at a table of values:*

5.6	5
5.9	6
6.2	7
6.5	8
6.8	9
7.1	10

*When the slope is found by using any two points in the table, it is also  $\frac{10}{3}$  as was indicated in the equation shown in the previously mentioned second possible student approach.*



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*One example of this is shown below using the two points (5.6, 5) and (7.1, 10).*

$$\frac{(y - y_1)}{(x - x_1)} = \frac{10 - 5}{7.1 - 5.6}$$

$$= \frac{5}{1.5}$$

$$= \frac{50}{15}$$

$$= \frac{10}{3}$$

*From this result, we know  $y = \frac{10}{3}(x - x_1) + y_1$  by using the point-slope form introduced during the “Forget the Formula” task.*

*Substituting one of the points in the table shown above (5.6, 5) for  $(x_1, y_1)$  it is seen that  $y = \frac{10}{3}(x - 5.6) + 5$*

*or  $y = \frac{10}{3}x + \left(\frac{10}{3} \cdot \frac{-56}{10}\right) + 5$ .*

*This leads to  $y = \frac{10}{3}x + \left(\frac{-56}{3} + \frac{15}{3}\right)$  or  $y = \frac{10}{3}x + \frac{-41}{3}$ .*

*Another approach could be to use the slope in the more popular y-intercept form.*

*Using one of the points in the table to calculate  $b$ , also leads to the value of  $\frac{-41}{3}$  shown above.*

*The example below is substituting the values (5.9, 6) for  $(x, y)$ .*

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$$\begin{aligned}y &= mx + b \\6 &= \left(\frac{10}{3}\right)(5.9) + b \\6 &= \frac{59}{3} + b \\b &= \frac{18 - 59}{3} \\b &= \frac{-41}{3}\end{aligned}$$

*Teachers should encourage students to approach problem-solving from multiple directions. This will help them to develop understandings concerning the relationships of the various forms of writing linear equations.*

- **Right To the Point Learning Task**

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### Right To the Point

Alex drew a circle with its center at the origin. The point (3, 4) is on his circle. He constructed a line that is perpendicular to the radius of the circle through that point. Since Alex knows you are studying about equations of lines in your mathematics class, he wants you to find the equation of the line that he constructed. Write an explanation for Alex about how you found the equation.

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### *Discussion, Suggestions, Possible Solutions*

*Students may be interested in knowing that a line perpendicular to the radius through a point on a circle is called a tangent line and they will be studying more about this in high school. Students may develop the understanding that the slopes of perpendicular lines are negative reciprocals through this task. They may do this by actually constructing the line and using points on the line to find its slope.*

*The task involves finding the equation of a line with a slope of  $-\frac{3}{4}$  and which contains the point (3, 4). In slope-intercept form, the equation will be  $y = -\frac{3}{4}x + b$ , since the coefficient of  $x$  is the slope of the line. All that remains is to find the value of  $b$  which satisfies the equation  $4 = -\frac{3}{4}(3) + b$ , since (3, 4) is a point on this line. Solving this equation yields that  $b = \frac{25}{4}$ , so the equation of the line perpendicular to the radius of the circle at the given point (the point of tangency) is  $y = -\frac{3}{4}x + \frac{25}{4}$ .*

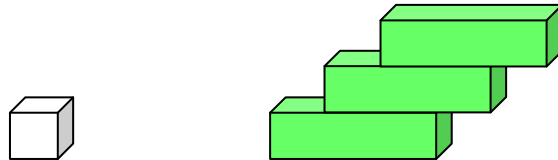
- **Staircases**

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### Staircases

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A same-color staircase is made from Cuisenaire rods. Each time that a rod is added to the staircase, it is offset by the space of a white (unit) rod. The rods that are used to make the staircase are 3 units in length.

- A. What is the surface area and volume of a staircase that is 3 units tall?
- B. Predict the volume and surface area of a staircase that is 5 units high. Find the actual surface area and volume, and compare them with your answer. Explain any discrepancies that you found.
- C. What will the volume and surface area be when you add the hundredth rod? Explain how you found your answer.
- D. Develop a general method or rule that can be used to determine the volume and surface area for any number of rods. Explain your thinking.
- E. How would changing the color of the rods affect your results? Verify your answer by completing A-D for a different color. Were the results what you predicted? If not, explain where your thinking was off.
- F. How would changing the way the stairs were made affect your results? For example, allowing different colors of rods to be used to make the stairs. Describe how you would change the way the stairs were made and complete A-D with this new pattern. Were the results what you predicted? If not, explain where your thinking was off.

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### *Discussion, Suggestions, Possible Solutions*

- A. What is the surface area and volume of a staircase that is 3 units tall?

*Students should have become familiar with working tasks in context. This leaves this task open to various interpretations concerning surface area. Regardless of the interpretation, students should work the entire task being consistent with their interpretation. Some students may think of painting the surface, some may consider carpeting stairs, and others may visualize a mold for pouring concrete. Each of these situations could yield different results and as long as the students are able to justify their reasoning and are mathematically correct, they should be permitted to have these interpretations. During the sharing time, excellent discussions could open opportunities for deeper understandings with a variety of contexts.*

*If all surfaces are painted (including the bottom), the surface area of a staircase that is 3 units tall could be found by (top + bottom + front + back + left side + right side). The top and bottom are each  $(1 + 1 + 3)$ , the front and back are each  $(3 \cdot 3)$ , and the sides are each  $(3 \cdot 1)$ . Thus, the surface area of a staircase that is 3 units tall would be  $2(5) + 2(9) + 2(3) = 34$  square units.*

*The volume of the staircase that is 3 units tall would be  $3(3) = 9$  cubic units.*

- B. Predict the volume and surface area of a staircase that is 5 units high. Find the actual surface area and volume, and compare them with your answer. Explain any discrepancies that you found.

*Some students may be able to see a pattern this early in the task and notice that each rod will add 10 square units to the surface area of the previous staircase. Therefore, the surface area of a staircase that is 5 units high will add 20 square units to the staircase that is 3 units high, or 54 square units. Not all students will see this until they have worked more with the task.*

*The volume of the staircase that is 5 units tall would be  $3(5) = 15$  cubic units.*

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- C. What will the volume and surface area be when you add the hundredth rod?

*Students may have different approaches. However, many of them may see the pattern that was mentioned in part B above. This means that there are 95 rods added to the staircase in part B, yielding  $95(10) + 54$  square units or 1004 square units for a staircase that is 100 rods tall.*

*The volume of the staircase that is 100 units tall will be  $3(100) = 300$  cubic units.*

- D. Develop a general method or rule that can be used to determine the volume and surface area for any number of rods. Explain your thinking.

*Again, students may not only have different interpretations for surface area, they also may use a variety of different strategies and approaches for developing their general rule. Regardless of the strategies, those groups that have the same interpretations, should end up with the same rule although the rules may look quite different. Take advantage of opportunities for students to develop their understanding of equivalent equations and expressions.*

*One example of approaching this could be to see that the top and bottom rods have the same surface area of 12 square units. As mentioned earlier, each of the middle rods will have a surface area of 10 because of the overlap. Therefore, a general rule could be expressed in words as two times the surface area of the top (or bottom) added to ten times the number of additional rods will yield the surface area of any number of stairs. Algebraically, this could be expressed as  $SA = 2(12) + (n - 2)(10)$  where  $n$  = the total number of rods. This may be simplified as  $SA = 24 + 10n - 20$  or  $SA = 10n + 4$ . Understand that there are MANY other approaches that students may use to arrive at this general rule.*

*The volume of any staircase that is  $n$  units tall will be  $3n$  cubic units.*

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*The next two parts of the task are usually enjoyed by most students. However, some students take more time to complete the previous parts. Therefore, students should not be punished if they do not have the time to complete all of the questions shown below. Every student should be given an opportunity to attempt them should they have the time. Answers will vary based on the creativity of the students.*

- E. How would changing the color of the rods affect your results? Verify your answer by completing A-D for a different color. Were the results what you predicted? If not, explain where your thinking was off.
- F. How would changing the way the stairs were made affect your results? For example, allowing different colors of rods to be used to make the stairs. Describe how you would change the way the stairs were made and complete A-D with this new pattern. Were the results what you predicted? If not, explain where your thinking was off.

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*A direct link to this article is [http://my.nctm.org/eresources/view\\_media.asp?article\\_id=6504](http://my.nctm.org/eresources/view_media.asp?article_id=6504).*

- **Crossing the River**

Georgia Performance Standards Framework for MATHEMATICS – GRADE 8

**Crossing the River.**

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Eight adults and two children need to cross a river. A small boat is available that can hold one adult, or one or two children.

There are three possibilities:

- 1) 1 adult in the boat
- 2) 1 child in the boat
- 3) 2 children in the boat

Everyone can row the boat.

- A. How many one-way trips does it take for them all to cross the river? Explain your thinking.
- B. Can you describe how to work it out for 2 children and any number of adults?
- C. Show how your rule works out for 100 adults?
- D. What happens to the rule if there are different numbers of children? For example: 8 adults and 3 children? 8 adults and 4 children?
- E. Write a rule for finding the number of trips needed for A adults and C children.
- F. If one group of adults and children took 27 trips. How many adults and children were in the group? Is there more than one solution? Why or why not?

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### *Discussion, Suggestions, Possible Solutions*

*You may want to allow your students to experience the following website to help with determining the fewest number of trips needed for smaller numbers of adults and children prior to experiencing this task.*

<http://pbskids.org/cyberchase/games/modeling/>

- A. How many one-way trips does it take for them all to cross the river? Explain your thinking.

*Some students will do better with this task when using manipulatives so that they may actually model the trips in the boat. The teacher may also want to assure that students understand the meaning of a “one-way” trip prior to beginning this task.*

Trip number	One side of the river	In the boat	Other side of the river
1	A <sub>1</sub> A <sub>2</sub> A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub>	→ C <sub>1</sub> C <sub>2</sub> ↓	
2	A <sub>1</sub> A <sub>2</sub> A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub>	↑↑ C <sub>1</sub> ↑	C <sub>2</sub>
3	A <sub>2</sub> A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub> C <sub>1</sub>	→ A <sub>1</sub> ↓	C <sub>2</sub>
4	A <sub>2</sub> A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub> C <sub>1</sub>	↑↑ C <sub>2</sub> ↑	A <sub>1</sub>
5	A <sub>2</sub> A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub>	→ C <sub>1</sub> C <sub>2</sub> ↓	A <sub>1</sub>
6	A <sub>2</sub> A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub>	↑↑ C <sub>1</sub> ↑	A <sub>1</sub> C <sub>2</sub>
7	A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub> C <sub>1</sub>	→ A <sub>2</sub> ↓	A <sub>1</sub> C <sub>2</sub>
8	A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub> C <sub>1</sub>	↑↑ C <sub>2</sub> ↑	A <sub>1</sub> A <sub>2</sub>
9	A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub>	→ C <sub>1</sub> C <sub>2</sub> ↓	A <sub>1</sub> A <sub>2</sub>
10	A <sub>3</sub> A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub>	↑↑ C <sub>1</sub> ↑	A <sub>1</sub> A <sub>2</sub> C <sub>2</sub>
11	A <sub>4</sub> A <sub>5</sub> A <sub>6</sub> A <sub>7</sub> A <sub>8</sub> C <sub>1</sub>	→ A <sub>3</sub> ↓	A <sub>1</sub> A <sub>2</sub> C <sub>2</sub>

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12	$A_4A_5A_6A_7A_8C_1$	$\uparrow\uparrow C_2 \uparrow$	$A_1A_2A_3$
13	$A_4A_5A_6A_7A_8$	$\rightarrow C_1C_2 \downarrow$	$A_1A_2A_3$
14	$A_4A_5A_6A_7A_8$	$\uparrow\uparrow C_1 \uparrow$	$A_1A_2A_3C_2$
15	$A_5A_6A_7A_8C_1$	$\rightarrow A_4 \downarrow$	$A_1A_2A_3C_2$
16	$A_5A_6A_7A_8C_1$	$\uparrow\uparrow C_2 \uparrow$	$A_1A_2A_3A_4$
17	$A_5A_6A_7A_8$	$\rightarrow C_1C_2 \downarrow$	$A_1A_2A_3A_4$
18	$A_5A_6A_7A_8$	$\uparrow\uparrow C_1 \uparrow$	$A_1A_2A_3A_4C_2$
19	$A_6A_7A_8C_1$	$\rightarrow A_5 \downarrow$	$A_1A_2A_3A_4C_2$
20	$A_6A_7A_8C_1$	$\uparrow\uparrow C_2 \uparrow$	$A_1A_2A_3A_4A_5$
21	$A_6A_7A_8$	$\rightarrow C_1C_2 \downarrow$	$A_1A_2A_3A_4A_5$
22	$A_6A_7A_8$	$\uparrow\uparrow C_1 \uparrow$	$A_1A_2A_3A_4A_5C_2$
23	$A_7A_8C_1$	$\rightarrow A_6 \downarrow$	$A_1A_2A_3A_4A_5C_2$
24	$A_7A_8C_1$	$\uparrow\uparrow C_2 \uparrow$	$A_1A_2A_3A_4A_5A_6$
25	$A_7A_8$	$\rightarrow C_1C_2 \downarrow$	$A_1A_2A_3A_4A_5A_6$
26	$A_7A_8$	$\uparrow\uparrow C_1 \uparrow$	$A_1A_2A_3A_4A_5A_6C_2$
27	$A_8C_1$	$\rightarrow A_7 \downarrow$	$A_1A_2A_3A_4A_5A_6C_2$
28	$A_8C_1$	$\uparrow\uparrow C_2 \uparrow$	$A_1A_2A_3A_4A_5A_6A_7$
29	$A_8$	$\rightarrow C_1C_2 \downarrow$	$A_1A_2A_3A_4A_5A_6A_7$
30	$A_8$	$\uparrow\uparrow C_1 \uparrow$	$A_1A_2A_3A_4A_5A_6A_7C_2$
31	$C_1$	$\rightarrow A_8 \downarrow$	$A_1A_2A_3A_4A_5A_6A_7C_2$
32	$C_1$	$\uparrow\uparrow C_2 \uparrow$	$A_1A_2A_3A_4A_5A_6A_7A_8$
33		$\rightarrow C_1C_2 \downarrow$	$A_1A_2A_3A_4A_5A_6A_7A_8$
			$A_1A_2A_3A_4A_5A_6A_7A_8C_1C_2$

*From the table, students should notice that it takes 31 trips for all eight of the adults to make it to the other side of the river. It takes an additional 2 trips to have both children on the other side of the river. This gives a total of 33 one-way trips for 8 adults and 2 children to cross the river.*

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- B. Can you describe how to work it out for 2 children and any number of adults?

*Prior to asking this question, students should understand that you are not looking for a “yes/no” answer. They should be instructed to actually describe how this may work for 2 children and any number of adults.*

*Using the information gathered from the chart above, and letting  $T$  = the number of trips needed, one could deduce that  $T = (4A - 1) + 2$  for any number of adults and 2 children to cross the river. It is suggested that students test this conjecture to feel confident that it really works every time.*

- C. Show how your rule works out for 100 adults?

*This would mean that  $T = 4(100) - 1 + 2$ . In other words, 401 trips would be required to have 100 adults and 2 children cross the river.*

- D. What happens to the rule if there are different numbers of children? For example: 8 adults and 3 children? 8 adults and 4 children?

*By expanding the chart shown above, if there are 8 adults, the first 31 trips would still be required to have all eight of the adults reach the other side of the river. The only difference up to that point on the chart would be that more children would be waiting on the original side of the river.*

Trip number	One side of the river	In the boat	Other side of the river
32	$C_1C_3$	$\uparrow C_2 \uparrow$	$A_1A_2A_3A_4A_5A_6A_7A_8$
33	$C_3$	$\rightarrow C_1C_2 \downarrow$	$A_1A_2A_3A_4A_5A_6A_7A_8$
34	$C_3$	$\uparrow C_1 \uparrow$	$A_1A_2A_3A_4A_5A_6A_7A_8C_2$

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35		$\rightarrow C_1 C_3 \downarrow$	$A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 C_2$
			$A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 C_1 C_2 C_3$

*This shows that two more trips were needed for one more child.*

*To see if this is always the case, students may want to use the manipulatives or expand the chart even more.*

Trip number	One side of the river	In the boat	Other side of the river
36	$C_4$	$\uparrow\uparrow C_3 \uparrow\uparrow$	$A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 C_1 C_2$
37		$\rightarrow C_3 C_4 \downarrow$	$A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 C_1 C_2$
			$A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 C_1 C_2 C_3 C_4$

- E. Write a rule for finding the number of trips needed for A adults and C children.

*This gives us a pattern that verifies  $T = (4A - 1) + 2(C - 1)$  with the requirement of at least one child, and may be simplified to  $4A + 2C = T + 3$ . This is an excellent example of using the Standard form for the equation of a line ( $ax + by = c$ ) and offers a great opportunity for discussion comparing and contrasting this form with the popular slope-intercept form.*

- F. If one group of adults and children took 27 trips. How many adults and children were in the group? Is there more than one solution? Why or why not?

*Using the above rule,  $T = (4A - 1) + 2(C - 1)$  and remembering that a minimum of 1 child is*

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*required, we find the possible results below.*

A	C	$(4A-1) + 2(C-1)$
14	1	$55 + 0 = 55$
13	2	$51 + 2 = 53$
12	3	$47 + 4 = 51$
11	4	$43 + 6 = 49$
10	5	$39 + 8 = 47$
9	6	$35 + 10 = 45$
8	7	$31 + 12 = 43$
7	8	$27 + 14 = 41$
6	9	$23 + 16 = 39$
5	10	$19 + 18 = 37$
4	11	$15 + 20 = 35$
3	12	$11 + 22 = 33$
2	13	$7 + 24 = 31$
1	14	$3 + 26 = 29$

*There is not only a requirement for at least one child to make any trip as discovered earlier, but to have exactly 27 trips,  $1 \leq A \leq 14$  must also be true.*

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- **What Would that Graph Look Like? Learning Task**

**What Would that Graph Look Like? Learning Task**

In math class one day Mrs. Dees conducted an experiment. In the first part of the experiment her students wrote  $x$ 's on a sheet of paper and counted how many they were able to make in one minute. Then Mrs. Dees told them to change their pencils to their other hands, and repeat the experiment. Mrs. Dees then gathered from each student the information about how many  $x$ 's were made with his/her right hand and how many were made with his/her left hand in one minute. Using the data for the right hand as the  $x$  values and the data for the left hand as the  $y$  values, explain what you expect a graph of this data would look like.

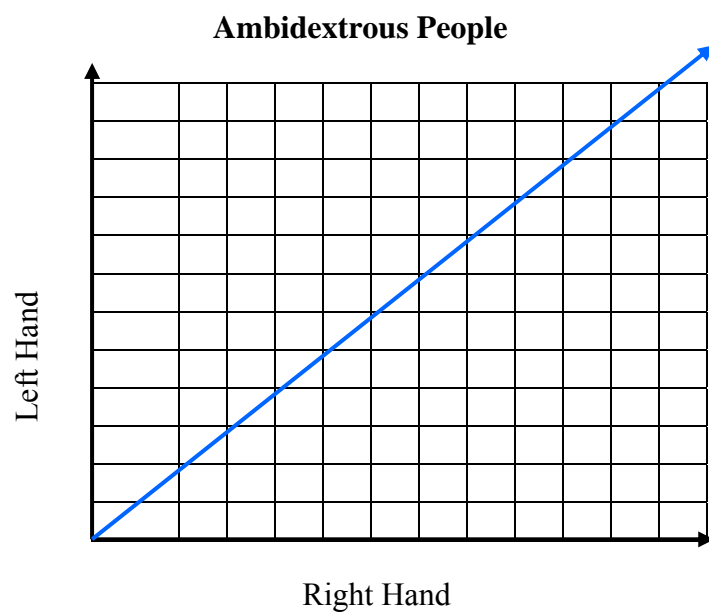
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***Discussion, Suggestions, Possible Solutions***

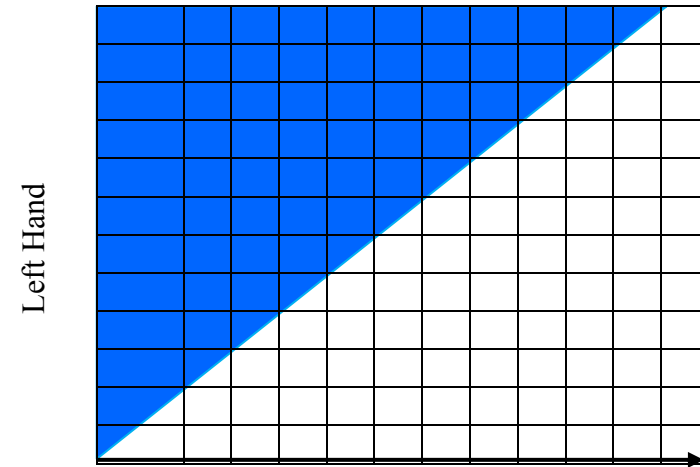
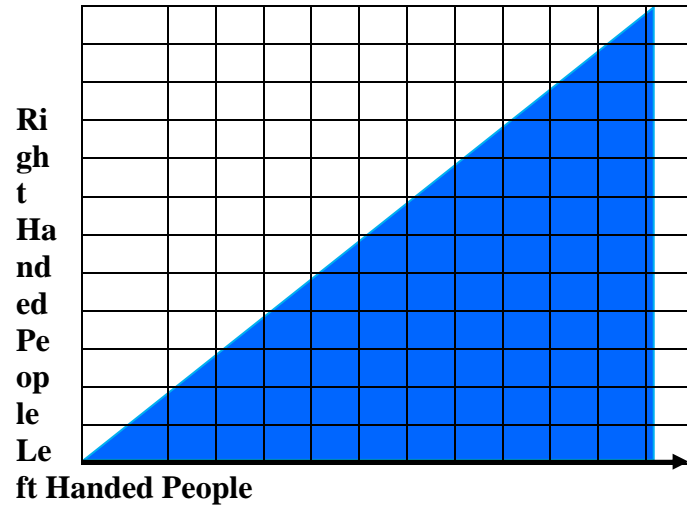
*In this context, a point on the line  $y = x$  would represent data for an ambidextrous person (a person that works equally as well with their left hand as they do their right hand. Left-handed persons would be able to make more  $x$ 's with their left hands than with their right hands in one minute, so their data would lie above the line  $y = x$ . Since most students are right-handed, most of the points would be graphed below the line  $y = x$ .*

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*This activity could be used to introduce graphing inequalities on the coordinate plane.*



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Right Hand

Right Hand

*Using a graphing calculator is an efficient way to capture the picture of the actual data. Enter the  $x$  values in List 1 ( $L_1$ ), the  $y$  values in List 2 ( $L_2$ ), and enter  $y_1 = x$ . Turn the statplot on to view the data in Lists 1 and 2, select Zoom Stat to set the viewing window appropriately, and select  $y_1$  to see the boundary line  $y = x$  along with the individual data points.*

- **Cholesterol – Good or Bad?**

**Cholesterol – Good or Bad?**

Matt's mom goes to the doctor regularly and has her cholesterol checked. The blood chemistry report shows several measurements related to cholesterol. Matt learned that there is a good kind of cholesterol (HDL) and a bad kind of cholesterol (LDL). In addition to being concerned about the total amount of cholesterol, people have to be concerned about the ratio of total cholesterol to good cholesterol. The average ratio of total to good cholesterol is 4.5 to 1. A ratio above 4.5 to 1 is an increased risk for heart disease.

Make a graph to help Matt see the combinations of total cholesterol and good cholesterol readings that would be higher-than-average risks. Let the  $x$  values represent the good cholesterol reading and the  $y$  values

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represent the total cholesterol readings.

Matt's mother's HDL = 35 and her total cholesterol was 200. His father had HDL = 60 and total cholesterol of 240. Help Matt to understand whether or not his parents' have combinations that are considered higher-than-average risks as far as the ratio of total to good cholesterol is concerned. Justify your answers.

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### ***Discussion, Suggestions, Possible Solutions***

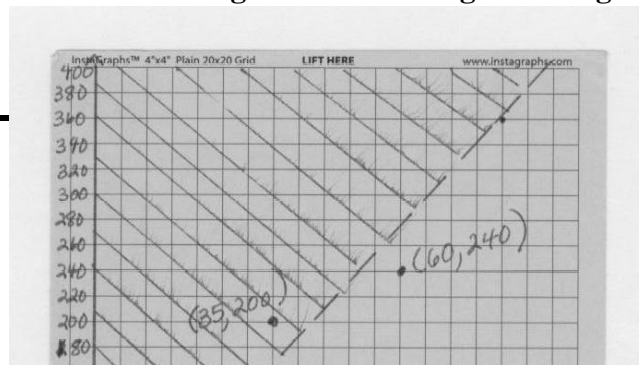
*Since the only data to be graphed is that representing higher-than-average ratios, the half-plane will be open. In order to find the boundary of the half-plane, find values of  $x$  and  $y$  that satisfy  $\frac{y}{x} = 4.5$ , which is equivalent to  $y = 4.5x$ . Some possible points to use are (10, 45), (20, 90), and (80, 360). These points are connected with a dotted (or dashed) line to indicate that they are not a part of the solution set. Points above that line are in the solution set of the inequality, so that half-plane is shaded.*

*To determine that (35,200) does represent a combination with a higher-than-average ratio, the student might observe that it is in the half-plane which was graphed. Also, the student could directly calculate the ratio of 200 to 35 as simplifying to approximately 5.7 to 1, which is above 4.5.*

*Similarly the student may observe that (60, 240) is not in the shaded half-plane, so it does not represent values with a high ratio. The ratio of 240 to 60 simplifies to 4 to 1, which is less than 4.5.*

*After the unit on systems, the class could return to this problem situation and show simultaneously the ratio information and the desirable totals for good and total cholesterol. Desirable total cholesterol levels are less than 200. Desirable HDL levels are greater than 40.*

### **Cholesterol Higher-than-Average Readings**



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Total Cholesterol

HDL

- **Them Bones**

**Them Bones**

Have you ever wondered how an archaeologist can tell from one ancient human bone, how tall the man or woman was when they were walking upon the earth?

In this task you will make predictions concerning the heights of people based upon the length of their humerus (the long bone in the arm stretching from the shoulder to the elbow) and the radius (the forearm bone on the thumb side).

Part I:

- Measure your height in centimeters.

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- Measure your humerus bone in centimeters.
- Measure your radius bone in centimeters.

### Part II:

- If you are a male, collect and organize all three of these measurements from each of the male students in your class.
- If you are a female, collect and organize all three of these measurements from each of the female students in your class.
- Label two identical first quadrant graphs allowing the  $x$ -axis to represent bone lengths and the  $y$ -axis to represent heights.

### Part III:

- On the first graph, plot your data using the humerus bone lengths and student heights.
- On the second graph, plot your data using the radius bone lengths and student heights.

### Part IV:

- For both of the graphs, use something similar to a strand of spaghetti to represent a line. Position the ‘line’ so that the plotted points are as close to it as possible to determine a ‘line of best fit’ or ‘trend line’.
- Find the equation of this line for both the “Humerus/Height” graph and the “Radius/Height” graph. Describe in detail how you determined these two lines.
- Compare the slopes of the two lines of best fit and explain the real-world meaning of these slopes.
- Explain what the  $y$ -intercepts represent.

### Part V:

- Predict the height of a person of your gender that had a humerus bone with a length of 18cm using your data. Show how you arrived at your prediction.
- Predict the length of the radius bone of a person of your gender that was 145cm tall using your data. Show how you arrived at your prediction.

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### *Discussion, Suggestions, Possible Solutions*

#### Part I:

- Measure your height in centimeters.
- Measure your humerus bone in centimeters.
- Measure your radius bone in centimeters.

*You may wish to have students work with a partner or small group to make it easier for the measurements to be accurate. The partners or small groups should be of the same gender.*

#### Part II:

- If you are a male, collect and organize all three of these measurements from each of the male students in your class.
  
- If you are a female, collect and organize all three of these measurements from each of the female students in your class.
- Make two identical first quadrant graphs allowing the  $x$ -axis to represent bone lengths and the  $y$ -axis to represent heights.

*Students should have prior knowledge of how to collect and organize their data. Be sure that they include their own measurements in their data. You may want the students to make three identical graphs instead of two if you feel they will understand parts IV and V better with the lines of best fit placed on a graph together without the rest of the scatter plot points.*

#### Part III:

- On the first graph, plot your data using the humerus bone lengths and student heights.
- On the second graph, plot your data using the radius bone lengths and student heights.

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*In Grade 7, students became familiar with scatter plots. They should be able to develop their own graphs on grid paper without the teacher needing to supply previously made graphs. They may want to discuss best ways to label the axis of each graph with their partner or small group prior to making the graphs.*

Part IV:

- For both of the graphs, use something similar to a strand of spaghetti to represent a line. Position the ‘line’ so that the plotted points are as close to it as possible to determine a ‘line of best fit’ or ‘trend line’.
- Find the equation of this line for both the “Humerus/Height” graph and the “Radius/Height” graph. Describe in detail how you determined these two lines.
- Compare the slopes of the two lines of best fit and explain the real-world meaning of these slopes.
- Explain what the y-intercepts represent.

*It is not necessary to use a piece of spaghetti. However, it is usually easier for students to visualize a line of best fit using an item that is thin and straight to ‘see’ how close it represents the majority of the points on a scatter plot. A line of best fit may pass through all of the points, some of the points, or none of the points. Regardless, it should best represent the data.*

*To develop an equation for a line of best fit, students should pick two points that fall on the line to use in finding the slope of the line. From this, they can use the slope-intercept form as an easy way to find the y-intercept.*

*It is possible that students will choose different points and their equation will be slightly different. Student answers should be considered CORRECT, as long as their calculations are correct for the points that were picked to develop the line. This means it is imperative that students show their work and explain their thinking.*

*Students may ask, “Who has the REAL line of best fit?” Let them know that as they move into future*

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*mathematics courses, they will work with regression and be able to become more exact when determining lines of best fit. At this point and time in their learning of mathematics, the purpose of the line of best fit will be used to make predictions. They should understand that predictions are simply a forecast, guess, or calculation and may vary slightly from an exact result.*

Part V:

- Predict the height of a person of your gender that had a humerus bone with a length of 18cm using your data. Show how you developed your prediction.
- Predict the length of the radius bone of a person of your gender that was 145cm tall using your data. Show how you developed your prediction.

*When data is displayed with a scatter plot, it is often useful to attempt to represent that data with the equation of a straight line for purposes of predicting values that may not be displayed on the plot as mentioned above.*

*As an extension, students may be interested to know that when predicting, they may be interpolating or extrapolating. When looking for values that fall within the plotted values, they are **interpolating**. When looking for values that fall outside the plotted values, they are **extrapolating**. They should understand that when extrapolating, the further away from the plotted values they go, the less reliable their prediction will be. You may want to challenge them with values that fit both categories.*

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- **Bungee Jump**

### **Bungee Jump**

Your job is to make a bungee cord to allow for the most exciting, but safe, jump as possible. To model this situation, use rubber bands and a toy. Use what you know about collecting data to decide how to predict the number of rubber bands needed for your toy to “jump” a distance that will be given to you later.

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#### ***Discussion, Suggestions, Possible Solutions***

*The teacher should have plenty of rubber bands, measuring tapes, and toys (we used bean bags) ready for each group to use during this task.*

*Students should collect their data and each group should explain to the class how they will determine the number of rubber bands they will use before the teacher announces the “jump” distance.*

*Measuring tapes should be removed prior to the announcement of the distance and students should understand that there are no “practice jumps”. An added thrill for the students is to have the “jump” distance pre-measured in another location within the school. When all toys are prepared (all bungee cords attached), the class may move to this location. Students enjoy having one representative from each group as a “set of judges” that measure and record the results of each “jump”. Certificates for the group that has the most thrilling experiment (gets the closest to the floor without “crashing”).*



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*Good discussions are likely to occur during this experiment such as how temperature, prior use, varied weights and other factors may influence the bungee cords.*

- **Culminating Task**

This culminating task represents the level of depth, rigor and complexity expected of all 8<sup>th</sup> grade students to demonstrate evidence of learning.

**Unit Five Task: “Is the Data Linear”**

Several experiments are described below.

Choose as many of them as time permits for collection and analysis of data.

Before performing each experiment, make a conjecture as to whether you believe the data collected will represent a linear relationship between the variables. After collecting your data, use more than one method to analyze whether a linear model is a good fit. For experiments which you believe to exhibit a linear trend, find the equation for the line of best fit and interpret the meaning of the coefficients in the problem context, if possible. Also use your predictor equations to answer questions you pose dealing with each context. For each experiment that does not follow a linear pattern, explain as much as you can about the relationship between the variables.

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**(1) Rolling Cars**

Make an incline (ramp) using a stack of books. Choose a particular car to use from the assortment of small toy cars. Release the car at the top of the ramp. Measure how many centimeters the car rolls when released from various heights. You could measure the height in terms of the number of books used or measure the height in centimeters.

**(2) Spring Experiment**

For this experiment you will need a spring and several weights the same size. Attach the spring to something stationery (overhead projector handle, doorknob, hook, etc.). On the other end of the spring attach a large paper clip or other device for hanging the weights. Measure the length of the spring with 0, 1, 2, 3, and 4 weights attached.

**(3) Candy Experiment**

On a paper napkin or paper plate pour a supply of candies that have a letter marked on one side. Count the beginning number of candies. Shake the candies in a bag and pour them back on the napkin or plate. Remove (and eat) any of the candies that have the letter showing on the top side. Count how many are left and record your data. Repeat the shaking, eating, counting, and recording steps until you have one or zero candies left. Your data will be comparing the number of candies left to the number of shakes.

**(4) Bouncing Ball**

Your group will need one ball and a meter stick. Record for various heights that the ball is dropped, how many centimeters the ball bounces back up. For example, one group member may hold the meter stick and drop the ball from a position that is 90 centimeters from the floor. Another group member would watch closely to measure the

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height to which the ball bounces.

(5) Meter Stick Experiment

Your group will need three meter sticks. One meter stick is to be placed in various positions with one end against a wall, so that it reaches different heights up the wall. For example, the group may place the meter stick so that it reaches 60 centimeters up the wall. Next the group measures how far the other end of the meter stick (which is against the floor) is from the wall. Record the data for comparing corresponding measurements for the distance the meter stick is from the wall and the distance the meter stick is from the floor.

**Suggestions for Classroom Use**

While this task may serve as a summative assessment, it also may be used for teaching and learning. It is important that all elements of the task be addressed throughout the learning process so that students understand what is expected of them.

- Peer Review
- Display for parent night
- Place in portfolio
- Photographs

**Discussion, Suggestions and Possible Solutions**

*In the culminating task, students are not told whether the relationships will be linear. They might expect each one to be a linear function simply because that is the topic they are studying. However, the candy experiment and the meter stick experiment do not represent linear functions.*

*The candy experiment represents an exponential function, where the number of candies can be estimated by multiplying the previous number by one-half each time the candies are shaken and poured out. The number left is not obtained by subtracting the same amount each time (a constant rate of change), but rather by multiplying by the*

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*same factor each time. This activity will help students distinguish between non-linear and linear functions and prepare them for a more formal study of exponential functions later in mathematics.*

*The meter stick experiment can be used to revisit the Pythagorean Theorem. In the experiment the meter stick serving as the hypotenuse is always 100 cm long. The height up the wall and the distance from the wall are the legs of a right triangle. Thus,  $x^2 + y^2 = 10,000$ , and  $y = \sqrt{10,000 - x^2}$ .*

*If the graphing calculator is being used, enter the data in two lists and do a statplot of the data. Then enter the theoretical model shown above in  $y =$  and view the data and the model simultaneously.*

*To include inequalities within this culminating task, students may be asked to look at the points that lie above the line of best fit and the points that lie below the line of best fit. Then have them show how they would represent algebraically the half-planes in which these points lie.*